Analytic Derivation of the Quark Distribution Functions Near x=1 by Statistical Methods

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Abstract

In this paper, we made a statistical approach on the thermodynamic structure of the nucleon and its quark distributions. We assume that the nucleon is a thermodynamic system of quarks and gluons. After we derived the quark density of states from the Dirac Equation, we calculated each quark's chemical potential, associating it with the quark's spin distribution. By using the sum rule of the longitudinal momentum, we obtain a function showing the relation between the temperature, radius and mass of the nucleon. The nucleon's radius is determined by the total energy minimal principle, and thus the temperature is also known. Finally, we deduced the quark distribution functions near x=1, and the results are in agreement with the experimental data.

1 Introduction

It has been justified by several authors in the literature that the nucleon, which confines many sea quarks and gluons, could be considered as a thermodynamic system with temperature T in a spherical volume with the radius R [1], [2], [3], [4], [5]. The statistical method based on the grand canonical ensemble formalism is then introduced to study the thermodynamic properties of the nucleon [6]. After R. S. Bhalerao found a relation between the quark distibution functions $q(x)$ in the infinite momentum frame and its density of states $\rho(E)$ in the nucleon rest frame [3],

$$q(x) = \frac{M^2 x}{2} \int_{Mx/2}^{M/2} g f(\mu) \rho(E) \frac{dE}{E^2}, \tag{1}$$

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where M is the nucleon mass, g is the quark degeneracy factor and $f(\mu)$ is the Fermi distribution function, the statistical methods can be used to calculate various distribution and structure functions of the nucleon [7].
However, in order to settle down all the parameters in eq (1), we have to find out the quark density of states and their chemical potentials at first. In this paper, we deduce the density of states by Dirac Equation and obtain the chemical potentials by inputting Δu, Δd, Δs. Our method differs from Bhalerao's model [7] in that:
1. We use a new density of states derived from D.E.;
2. We determine the nucleon's radius by the T-R-M function and the total energy minimal principle;
3. We use analytic methods, and our results are only about the $x \approx 1$ region. Futher results may able to be extented to small x region if we use numerical methods.

2 Dirac Equation and Quark Density of States

In our recently proposed Quark Gluon Coupling Model [6], we describe the nucleon as confining the quarks and gluons in a spherical volume with the radius of R. The quarks interact through the exchange of gluons, and the gluon couples to the conserved quark-current. The Lagrangian density for this field is

$$L = \overline{\psi}\left[\gamma^{\mu}\left(i\partial_{\mu} - g_c A_{\mu}\right) - m_q\right]\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{2}$$

where A_{μ} is the gluon vector field, g_c is the coupling constance, m_q is the quark mass and $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$. If we replace the vector field operators by their expectation values, that is, $A_{\mu} \to \delta_{\mu 0}A_0$, the Lagrange's equations yield the the linear Dirac Equation,

$$\left[i\gamma^{\mu}\partial_{\mu} - g_c\gamma^0 A_0 - m_q\right]\psi = 0. \tag{3}$$

The normalized quark wave function within a spherical volume with the radius of R is

$$\psi(\vec{r},t) = Ne^{-iEt} \times \begin{pmatrix} \sqrt{\frac{E-g_cA_0+m_q}{E}}j_0(\sqrt{(E-g_cA_0)^2 - m_q^2}r) \\ i\sqrt{\frac{E-g_cA_0-m_q}{E}}j_1(\sqrt{(E-g_cA_0)^2 - m_q^2}r)) \end{pmatrix} \frac{\chi_q}{\sqrt{4\pi}}, \tag{4}$$

where $r = |\vec{r}|$, χ_q is the quark spinor and N is the normalization constant [8] and E is the quark energy eigenvalue. We note that $\overline{\psi}\psi$ should vanish at the

boundary in the relativistic theory [9], $\overline{\psi}\psi\Big|_{r=R} = 0$. This yields the energy states [6]:

$$E_n \approx g_c A_0 + \sqrt{m_q^2 + (\frac{n\pi}{R})^2}, \tag{5}$$

where $n = 1, 2, ...$, and therefore the quark density of states is

$$\rho(E) = \frac{R(E - g_c A_0)}{\pi\sqrt{(E - g_c A_0)^2 - m_q^2}}. \tag{6}$$

If we use the assumption in the parton model that $m_q \to 0$, the density of states will be reduced to $\rho(E) = \frac{R}{\pi}$. We will use this density of states in all of the following sections, and we find the results are satisfactory.

3 Chemical Potential and Spin Distribution

In this section we will discuss the relation between quarks' chemical potentials and Δu, Δd, Δs, R. Many experiments show that we should take quarks with spin parallel to the nucleon's spin ($q \uparrow$) different than those with spin anti-parallel ($q \downarrow$). The main compoents of the nucleon, except for the gluons, are $u\uparrow$, $u\downarrow$, $d\uparrow$, $d\downarrow$, $s\uparrow$, $s\downarrow$ and their anti-particles $\bar{u}\downarrow$, $\bar{u}\uparrow$, $\bar{d}\downarrow$, $\bar{d}\uparrow$, $\bar{s}\downarrow$, $\bar{s}\uparrow$ [7]. We assume that heavy quarks c, b, t almost do not appear [3].
In the nucleon, there are 12 constraints on the numbers and chemical potentials of the above 12 types of quarks. If q stands for u, d, s quarks generally, Δq stands for $\Delta u, \Delta d, \Delta s$, n_q stands for valence quark number and μ_q stands for chemical potential, we have the constraints as follow:

$$n_{q\uparrow} - n_{\bar{q}\downarrow} = \frac{(n_q + \Delta q)}{2}; \tag{7}$$

$$n_{q\downarrow} - n_{\bar{q}\uparrow} = \frac{(n_q - \Delta q)}{2}; \tag{8}$$

$$\mu_{q\uparrow} + \mu_{\bar{q}\downarrow} = 0; \tag{9}$$

$$\mu_{q\downarrow} + \mu_{\bar{q}\uparrow} = 0. \tag{10}$$

By integrating eq (1) over x, we obtain that:

$$n_q = \int_0^{M/2} 3f(\mu_q)\rho(E)dE \tag{11}$$

Using the density of states deduced in section 2 and assuming that the nucleon's mass is much biger than μ_q, we get:

$$n_q = \frac{3RT}{\pi}\ln[\exp(\frac{\mu_q}{T}) + 1] \tag{12}$$

From eq (7), (8), (9), (10), (12), we get:

$$\mu_{q\uparrow} = \frac{(n_q + \Delta q)\pi}{6R}, \tag{13}$$

$$\mu_{q\downarrow} = \frac{(n_q - \Delta q)\pi}{6R}. \tag{14}$$

The relation between Δq and chemical potentials is:

$$\Delta q = \frac{3R(\mu_{q\uparrow} - \mu_{q\downarrow})}{\pi}. \tag{15}$$

From several experimental data [10], [11], [12], we use $\Delta u = 0.83$, $\Delta d = -0.43$, $\Delta s = -0.10$ for the proton and $\Delta u = -0.40$, $\Delta d = 0.86$, $\Delta s = -0.06$ for the neutron in the following sections.
Once the nucleon radius R is determined, all the chemical potentials are known from eq (13), (14), (9), (10). Even when R is unknown, we still can get three conclusions from those equations:
1. The ratios of each chemical potentials are: $\mu_{u\uparrow} : \mu_{u\downarrow} : \mu_{d\uparrow} : \mu_{d\downarrow} : \mu_{s\uparrow} : \mu_{s\downarrow}=$ 2.83 : 1.17 : 0.57 : 1.43 : -0.10 : 0.10 for the proton and $\mu_{u\uparrow} : \mu_{u\downarrow} : \mu_{d\uparrow} : \mu_{d\downarrow} : \mu_{s\uparrow} : \mu_{s\downarrow}=$ 0.60 : 1.40 : 2.86 : 1.14 : -0.06 : 0.06 for the neutron.
2. For both proton and neutron, $u\uparrow$, $u\downarrow$, $d\uparrow$, $d\downarrow$, $\bar{s}\downarrow$, $s\downarrow$ have positive chemical potentials while their anti-particles have negative chemical potentials.
3. The absolute value of the chemical potentials of $s\uparrow$, $s\downarrow$, $\bar{s}\uparrow$, $\bar{s}\downarrow$ are much smaller than others.

4 T-R-M Function of the Nucleon

In this section we will discuss the relation between nucleon's temperature, radius and mass and finally determine the value of R and T.
The longitudinal momentum fraction of each types of quarks could be obtained by integrating over x the eq (1) mutiplied x:

$$p_q = \frac{4}{3M}\int_0^{M/2} 3f(\mu_q)\rho(E)EdE. \tag{16}$$

Similarly, the longitudinal momentum fraction of the gluons is:

$$p_g = \frac{4}{3M}\int_0^{M/2} 16 f_B(0)\rho(E)EdE, \tag{17}$$

where $f_B(0)$ is the Bose distribution function and the chemical potential of the gluons is 0. Our calculation shows that for both proton and neutron, $p_{u\uparrow}$, $p_{u\downarrow}$, $p_{d\uparrow}$, $p_{d\downarrow}$ and p_g are relatively large, while $p_{s\uparrow}$, $p_{s\downarrow}$, $p_{\bar{s}\uparrow}$, $p_{\bar{s}\downarrow}$ are relatively small and $p_{\bar{u}\uparrow}$, $p_{\bar{u}\downarrow}$, $p_{\bar{d}\uparrow}$, $p_{\bar{d}\downarrow}$ are relatively tiny. From eq (16), (17), we have:

$$p_{u\uparrow} = \frac{\pi(n_u+\Delta u)^2}{18MR} + \frac{2\pi T^2 R}{3M}; \tag{18}$$

$$p_{u\downarrow} = \frac{\pi(n_u-\Delta u)^2}{18MR} + \frac{2\pi T^2 R}{3M}; \tag{19}$$

$$p_{d\uparrow} = \frac{\pi(n_d+\Delta d)^2}{18MR} + \frac{2\pi T^2 R}{3M}; \tag{20}$$

$$p_{d\downarrow} = \frac{\pi(n_d-\Delta d)^2}{18MR} + \frac{2\pi T^2 R}{3M}; \tag{21}$$

$$p_g = \frac{32\pi T^2 R}{9M}; \tag{22}$$

$$p_{s\uparrow} = p_{s\downarrow} = p_{\bar{s}\uparrow} = p_{\bar{s}\downarrow} = \frac{\pi T^2 R}{3M}; \tag{23}$$

$$p_{\bar{u}\uparrow} = p_{\bar{u}\downarrow} = p_{\bar{d}\uparrow} = p_{\bar{d}\downarrow} \approx 0, \tag{24}$$

where n_u and n_d are valence quark numbers. Note that the sum of all longitudinal momentum fractions is 1, that is,

$$p_{u\uparrow} + p_{u\downarrow} + p_{d\uparrow} + p_{d\downarrow} + p_g + p_{s\uparrow} + p_{s\downarrow} + p_{\bar{s}\uparrow} + p_{\bar{s}\downarrow} + p_{\bar{u}\uparrow} + p_{\bar{u}\downarrow} + p_{\bar{d}\uparrow} + p_{\bar{d}\downarrow} = 1. \tag{25}$$

From eq (18), (19), (20), (21), (22), (23), (24), (25) we get the T-R-M function as follows:

$$M = \frac{68\pi T}{9}\left(RT + \frac{5+\Delta u^2+\Delta d^2}{68RT}\right). \tag{26}$$

The total energy of the sphere E_T includes the zero point Casimir energy $\frac{C}{R}$, where $C \approx 2$:

$$E_T = M + \frac{C}{R} \tag{27}$$

We determine the nucleon radius R by the total energy minimal principle:

$$\left(\frac{\partial E_T}{\partial R}\right)_T = 0, \tag{28}$$

and therefore get:

$$R = \frac{2\pi(5 + \Delta u^2 + \Delta d^2 + \frac{9C}{2\pi})}{9M}, \tag{29}$$

$$T = \frac{9M\sqrt{(5 + \Delta u^2 + \Delta d^2 + \frac{9C}{\pi})}}{4\pi\sqrt{17}(5 + \Delta u^2 + \Delta d^2 + \frac{9C}{2\pi})}, \tag{30}$$

For the proton, if we use M=938 MeV, $\Delta u = 0.83$, $\Delta d = -0.43$, $C = 2$, we get $R = (154MeV)^{-1} = 1.28$ fm, $T = 63.5$ MeV. And for the neutron, if we use M=939 MeV, $\Delta u = -0.40$, $\Delta d = 0.86$, $C = 2$, we get $R = (153MeV)^{-1} = 1.29$ fm, $T = 63.4$ MeV. All chemical potentials can be obtained from eq (13), (14), (9), (10). Both the proton and neutron have similar radius and temperature, although the proton has a slightly smaller radius and higher temperature. We may take their radius and temperature the same in some approximations.

5 Quark Distribution Functions Near x=1

After we get the R, T and chemical potentials of each type of the quarks, from eq (1), we can obtain the quark distribution functions near x=1. We assume that x is so near 1 that $1 - x << \frac{2T}{M} \approx 0.135$. From eq (1), we have

$$q(x) = \frac{3RM(1-x)}{\pi x} \exp\left(\frac{\mu_q - Mx/2}{T}\right). \tag{31}$$

So that,

$$u(x) = u_\uparrow(x) + u_\downarrow(x) + \bar{u}_\uparrow(x) + \bar{u}_\downarrow(x)$$

$$= \frac{12RM(1-x)}{\pi x} \exp\left(\frac{-Mx}{2T}\right) \cosh\frac{n_u\pi}{6RT} \cosh\frac{\Delta u\pi}{6RT}; \tag{32}$$

$$\Delta u(x) = u_\uparrow(x) - u_\downarrow(x) + \bar{u}_\uparrow(x) - \bar{u}_\downarrow(x)$$

$$= \frac{12RM(1-x)}{\pi x} \exp\left(\frac{-Mx}{2T}\right) \sinh\frac{n_u\pi}{6RT} \sinh\frac{\Delta u\pi}{6RT}; \tag{33}$$

$$d(x) = d_\uparrow(x) + d_\downarrow(x) + \bar{d}_\uparrow(x) + \bar{d}_\downarrow(x)$$

$$= \frac{12RM(1-x)}{\pi x} \exp\left(\frac{-Mx}{2T}\right) \cosh\frac{n_d\pi}{6RT} \cosh\frac{\Delta d\pi}{6RT}; \tag{34}$$

$$\Delta d(x) = d_\uparrow(x) - d_\downarrow(x) + \bar{d}_\uparrow(x) - \bar{d}_\downarrow(x)$$

$$= \frac{12RM(1-x)}{\pi x} \exp\left(\frac{-Mx}{2T}\right) \sinh\frac{n_d\pi}{6RT} \sinh\frac{\Delta d\pi}{6RT}; \tag{35}$$

$$s(x) = s_\uparrow(x) + s_\downarrow(x) + \bar{s}_\uparrow(x) + \bar{s}_\downarrow(x)$$

$$= \frac{12RM(1-x)}{\pi x} \exp\left(\frac{-Mx}{2T}\right) \cosh\frac{\Delta s\pi}{6RT}; \tag{36}$$

$$\Delta s(x) = s_\uparrow(x) - s_\downarrow(x) + \bar{s}_\uparrow(x) - \bar{s}_\downarrow(x) = 0 \tag{37}$$

Thus, from eq (32), (33), (34), (35), (36), (37), we can calculate $\frac{\Delta u(x)}{u(x)}$, $\frac{\Delta d(x)}{d(x)}$, $\frac{\Delta s(x)}{s(x)}$, $\frac{d(x)}{u(x)}$, $\frac{s(x)}{u(x)}$, when $x \to 1$. For the proton, if we use the R, T in section 4, we have $\frac{\Delta u(x)}{u(x)} = 0.774$, $\frac{\Delta d(x)}{d(x)} = -0.425$ (quite in agreement with the quark-diquark model result [13], [14]), $\frac{\Delta s(x)}{s(x)} = 0$, $\frac{d(x)}{u(x)} = 0.216$, $\frac{s(x)}{u(x)} = 0.098$, when $x \to 1$. And for the Neuton, if we use the R, T in section 4, we have $\frac{\Delta u(x)}{u(x)} = -0.397$, $\frac{\Delta d(x)}{d(x)} = 0.786$, $\frac{\Delta s(x)}{s(x)} = 0$, $\frac{d(x)}{u(x)} = 4.83$, $\frac{s(x)}{u(x)} = 0.466$, when $x \to 1$. So that, when $x \to 1$, $F_2^n/F_2^p = 2.155u^n(x)/u^p(x) = 0.453$, which is quite in agreement with the recent experimental data [15] and the pQCD result [16], [17].

6 Conclusion

The analytic statistical method in this paper is limitted in the large x region. However, the advantage of this method is that it shows the relation between the quarks' chemical potentials and their spin distibutions, as well as the relation between the nucleon 's temperture, radius and mass, the so called T-R-M function. There are three input parameters, Δu, Δd, Δs, which cannot be determined without any dynamical approach beyond the statistical mechanics. Therefore the statistical method in this paper is only a connection between the phenomena, but it is helpful for our understanding.

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